Electromagnetic Levitation System Based on a PID Controller

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Abstract— The development of educative prototypes of industrial control techniques has increased in the past few years, since its importance lies in their low cost compared to real systems. This study presents the design, modeling, and implementation of a magnetic levitator. The complete system is formed by amplifiers, an optoelectronic sensor, a coil used as an electromagnet, and a metallic sphere used as an object to levitate. The information from the sensor is used to feed the proportional-integral-derivative (pid) controller, and with this regulate the incoming current that flows through the coil by means of a power amplifier with the objective of maintaining the metallic sphere at the balance point. The pid controller is designed based on dominating poles in a closed loop (s=-4 ± j4) being ±180°, assuring that the zeros contribute to the same angle. The proposed system levitates a metallic sphere with an inertial mass of 24.75 g and a diameter of 12 mm to a distance of 4mm.

Keywords—pid controller, electromagnet, magnetic levitation, dominating poles

I. Introduction

Magnetic levitation is a method by which an object is maintained afloat with a magnetic field that opposes gravitational force. Any object can be levitated if and when the magnetic field is strong enough. There are two techniques for doing this, repulsion and attraction. In magnetic repulsion, the induced currents in an electromagnet generate the magnetic force that opposes gravity, which allows a body to stay afloat. This system is stable on a vertical axis and has a natural balance point. In magnetic attraction, the body is adhered by means of magnetic flow generated by the induced currents that oppose gravity, this system is unstable without the help of control systems [1].

Some of the most common applications of magnetic levitation are Maglev trains, magnetic bearings, and the levitation of ornamental products such as plants and handcrafts [2]. On the other hand, the magnetic levitation system has attracted our attention in the industrial control field due to the challenge of stability, the relative grade of this system is not defined in some points of state space, and the system is also sub-actuated. For this reason, controller design and stabilizing the levitation of a mass of a specific weight is difficult. Controllers such as those of proportional, integral, and derivative (pid) action and the reverse-forward phase was designed to stabilize the levitation systems in an open loop. They were calculated with differential equations that control the dynamic of the linearized levitator close to the point of operation, creating controls to stabilize the system, which had small instabilities. [3].

II. Materials and Methods

This study implemented a magnetic levitation system composed of a coil actuated like an electromagnet, which levitated a metallic sphere with an inertial mass of 24.75 g placed below the coil at distance x. An important aspect for the coil design is that the diameter be 0.8 times smaller than the metallic sphere for optimal performance [4]. A commercial steel coil was used for the design of the coil (low in carbon) with a diameter of 9.5 mm and a longitude of 5 cm, the magnet wire is 22 caliber and has 500 coils.
The coil inductance changed due to the position of the sphere, it could be expressed analytically using an exponential function determined in equation (1). Where a is a constant distance, L_0 is the inductance when the sphere is not inside the field of action of the electromagnet, while L_l is the effect of the sphere when it is in contact with the electromagnet, x is the distance from the coil to the sphere.

$$L(x) = L_l + L_0 e^{\frac{-x}{a}} \tag{1}$$

Table 1 was obtained as a measuring reference of impedance and inductance of the coil with laboratory tests that varied distance x. This table is necessary to calculate the average a and a_p distance and is indispensable for the static balance of the sphere.

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Table 1. Gathering average distance a from the texts of the inductance obtained in the laboratory.

Distance (mm)	*Laboratorie measures (mH)*	*a calculated (mm)*	*a_p average (mm)*	*Mathematical adjustmetn with a_p (mm)*
0	35.8741	0.0000		35.8741
1	35.2888	2.5947		35.4711
2	35.0788	3.5074		35.1569
3	34.9438	4.2252		34.9118
4	34.7638	4.2861		34.7207
5	34.6438	4.4819		34.5717
6	34.5839	4.9140		34.4555
7	34.4789	4.8704		34.3649
8	34.4189	5.0451		34.2943
9	34.3589	5.1134	4.0203	34.2392
10	34.2990	5.0725		34.1962
11	34.2840	5.4133		34.1628
12	34.2540	5.5412		34.1366
13	34.2540	6.0030		34.1163
14	34.2090	5.8170		34.1004
15	34.1341	4.9786		34.0880
16	34.1041	4.6806		34.0783
17	34.0741	4.1347		34.0708
18	34.0591	3.7463		34.0649
19	34.0441	0.0000		34.0603
20	34.0441	0.0000		34.0568

In accordance with table 1, values were established for a = 4 mm, L_0= 1.8299 mH, y L_1 = 34.0441 mH.

The magnetic force that acts upon the metallic sphere is given with the equation (2):

$$f(x,i) = \frac{\partial W'}{\partial x} = -\frac{L_0}{2*a} i^2 e^{\frac{-x}{a}} \tag{2}$$

In static balance this force is balanced by the actuating gravitational force in the metallic sphere; when x=d e i = I, you get:

$$M * g = -\frac{L_0}{2a} i^2 e^{\frac{-d}{a}} = \frac{N^2 L_d}{2a} I^2 \tag{3}$$

Where N is the number of coil turns, L_d is the increase in inductance when $x = d$ due to the number of coil turns, g is the gravitational acceleration with an approximate value of 9.82 m/s^2 The current that circulates in the coil is obtained by isolating I from (3):

$$I = \sqrt{\frac{2Mga}{N^2 L_d}} = \sqrt{\frac{2Mga}{L_0 e^{\frac{-d}{a}}}} \tag{4}$$

A value of d = 5 mm is proposed, substituting the values of the equation (4), a value for I of 0.6086 A is obtained, which guarantees that the coil temperature is in a tolerable work range, since it will not exceed the current of the voltage source.

An optoelectronic sensor with a switch was used to detect the position of the metallic sphere over the coil. The information from the sensor is used to feed the Proportional-Integral-Derivative controller in order to regulate the incoming current I that flows in the coil through the power amplifier and maintain the metallic sphere at balance point$d$.

A. Mathematical Model

Due to the dynamic equations of the system not being linear, the current behaves as a I^2 parabola and because d is involved in operation point L_0, the equations should be linearized at operation point $x = d$ e $i = I$ (balance) which has been proposed.

In a balanced state, $Mg = f(I,d)$ is given with equation (3); applying the Laplace Transform to the Taylor series, and using equation (4), we can write it as:

$$\frac{X(s)}{I(s)} = -\frac{\frac{2Mg}{a}}{Ms^2 - \frac{Mg}{a}} = -\frac{M\frac{2g}{a}}{M\left(s^2 - \frac{g}{a}\right)} - \frac{\frac{2g}{a}}{s^2 - \frac{g}{a}} \tag{5}$$

Substituting the values of a and g in equation (5), the (plant) system transfer function is defined:

$$\frac{X(s)}{I(s)} = -\frac{\frac{2*9.82}{0.004}}{s^2 - \frac{9.82}{0.004}} = -\frac{21.4403}{s^2 - 2455} \tag{6}$$

This function links the coil current and the sphere position, which evaluates its position with the optoelectronic sensor, producing voltage proportional to the sphere position as a result. The sensor time constant is insignificant to that of the time of the levitator in the control system, for which the function of position sensor transfer simply converts into a constant.

$$X_0(s) = G_s X(s) \tag{7}$$

G_s represents the sensor gain, that for this system aims for 1205. From equations (6) and (7), the transfer function in which the distance functions as voltage is redefined:

$$\frac{X(s)}{I(s)} = -\frac{21.4403*1205}{s^2 - 2455} = -\frac{25835.6068}{s^2 - 2455} \tag{8}$$

This way, $X_0(s)$ represents the position of the levitating metallic sphere in the voltage function and $I(s)$ represents the electric current that goes through the coil.

In Figure 1, the diagram represents system blocks of electromagnetic levitation used in this study. The pid control maintains the position of the metallic sphere by means of a variation in current in the coil. The measuring element becomes the position variable in an adequate voltage variable, used to compare it with the incoming signal [5].

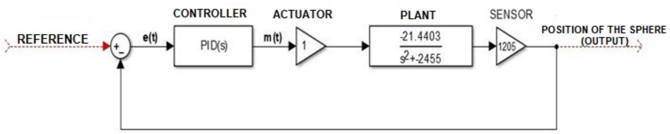

Fig. 1. Diagram of the blocks of the Magnetic Levitation System.

For the blocks of the integral and derivative actions of the controller, operational amplifiers were used, the same that give the previously calculated transfer function, according to the structure shown in Figure 1, signal$m(t)$ is the action of the pid controller, this acts on signal error e(t), given by equation 9:

$$m(t) = K_p e(t) + \frac{K_p}{T_i}\int_0^t e(t)dt + K_p T_d \frac{de(t)}{dt} \tag{9}$$

Applying the Laplace transform to equation 9, the transfer function of the pid controller in equation (10) is obtained:

$$M(s) = K_p E(s) + \frac{K_p}{T_i s} E(s) + K_p T_d E(s) s$$
$$M(s) = K_p E(s)\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$G_c(s) = \frac{M(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{10}$$

Where K_p is the proportional gain, T_i is the integrated time, T_d is the derivative time.
The configuration of the electronic pid cascade controller is formed with an adder and a proportional gain equalizer, integrated and derivative time with the following transfer function:

$$\frac{E(s)}{E_i(s)} = \frac{R_4(R_1C_1+R_2C_2)}{R_1R_3C_2}\left[1 + \frac{1}{(R_1C_1+R_2C_2)s} + \frac{R_1C_1R_2C_2}{(R_1C_1+R_2C_2)}s\right] \tag{11}$$

The equation (11) represents the transfer function of the pid controller to implement. Linking the equation (11) with (10), it is concluded that the proportional gain is:

$$K_p = \frac{R_4(R_1C_1+R_2C_2)}{R_1R_3C_2} \tag{12}$$

The integral time:

$$T_i = R_1C_1 + R_2C_2 \tag{13}$$

And for the derivative time:

$$T_d = \frac{R_1C_1R_2C_2}{(R_1C_1+R_2C_2)} \tag{14}$$

Locating the resulting poles for a system of the second order is given by (s + 0.7071± j0.7071) by a nominal frequency of 1 rad/s [6]. With this location of the dominating poles from the closed circuit, the suspension factor is calculated by obtaining $\delta = 0.7$ and a maximum overshoot of M_p= 4.5% from equation (15). An establishment time of $t_s = 1$ s is proposed, and according to the value of δ the natural unsuspended frequency is calculated in accordance with equation (16), obtaining $w_n = 5.714$ rad/s.

$$M_p(\%) = 100e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} \tag{15}$$

$$t_s = \frac{4}{\delta w_n} \tag{16}$$

The geometrical place of the roots gives us information about the transitory response and the frequency response, starting from the pole configuration and zeros of the system on the s plain. The roots of the characteristic equation are the values of 's' that fulfill the conditions of angle and magnitude. From Figure 1, the characteristic equation is defined as:

$$1 + G(s)H(s) = 0 \tag{17}$$

The values of the angles of the poles and zeros in the open circuit that make up the dominating poles of the closed circuit (s = -4 ± j4) should be ±180°, satisfying the determined specifications.
The two zeros from the controller at the location where both have the same angle, which is to say, they are located in the same place as plane 's', which is why the value of each angle should be 67.5º, considering this, the location of the two zeros is at s= -5.65 [7], Figure 2 represents the dominant poles in the 's' plane.

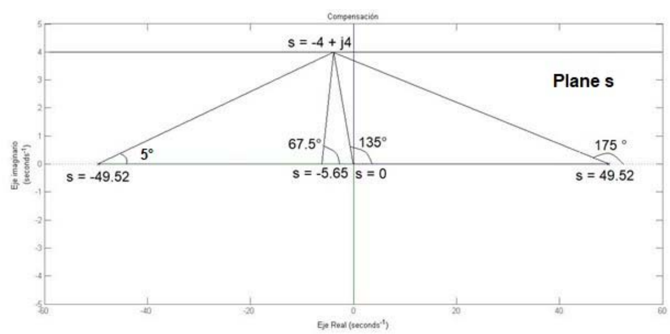

Fig. 2. Plane s for the location of zeros of the pid controller.

III. RESULTS AND DISCUSSION

Establishing the position of the zeros in the 's' plane, the transfer function of the pid controller is:

$$G_c(s) = \frac{K(s+5.65)^2}{s} \tag{18}$$

Subsequently, the following equation will be applied:

$$G_c(s)G_p(s)_{s=-4+j4} = 1 \tag{19}$$

with the objective of obtaining the gain with which the geometrical location of the roots pass through the desired point. Therefore:

$$\frac{25835.6068}{s^2-2455} * \frac{K(s+5.65)^2}{s} = 1 \tag{20}$$

Isolating K in (20), we have:

$$K = \frac{s(s^2-2455)}{25835.6068\,(s+5.65)^2}\ \ _{s=-4+j4} \tag{21}$$

$K = 0.02871$ Obtaining the value of K, the following is replaced in (18):

$$G_c(s) = \frac{0.02871(s+5.65)^2}{s}$$
$$G_c(s) = 0.02871[1 + \frac{1}{0.354s} + 0.0885\, s^2] \tag{22}$$

Linking equation (10) with equation (22), we find that the proportional gain is 0.02871, the integrative time is T_i= 0.354 s and the derivative time is T_d= 0.0885 s.
The link between (22) with (12), (13) and (14) was established, and the values of the pid controller were calculated to implement the cascade design in Figure 3.

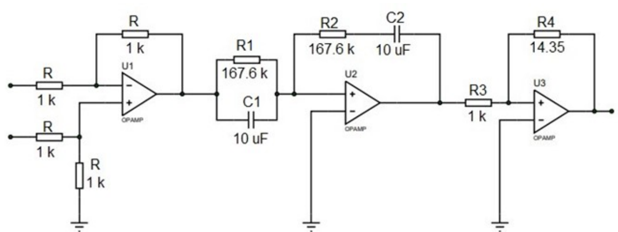

Fig. 3. Pid cascade circuit of the magnetic levitation system.

Setting values for C_1 y C_2 = 10 uF, and isolating R_1 y R_2 from equation 14, R_1 = R_2 = 167.685 kΩ is obtained. With the obtained values, R_3 = 1 kΩ is formulated and substitutes R_1, R_2, R_3, C_1 and C_2 in equation 12, therefore R_4 = 14.35 Ω.

A. System implementation

In Figure 4 the structure design of the levitation system is shown, using 1 inch PVC, this frame supports the coil and the optic sensor, the upper part has a wooden shelf that acts as a base for the pid controller circuit and the power circuit, as shown in Figure 5.

Fig. 4. Structure of the levitation system.

Fig. 5. PVC structure of the levitation system.

In Figure 6 the metallic sphere can be seen levitating an average distance of 4 mm from the coil, while in Figure 7 the weight of a washer has been added, verifying the established calculations for the pid controller for the object levitation trial.

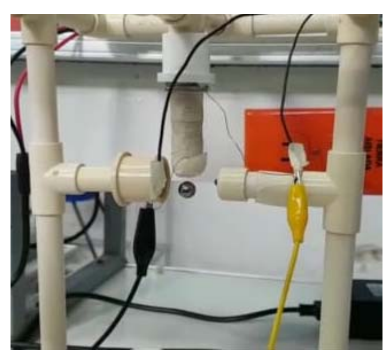

Fig. 6. Levitating sphere.

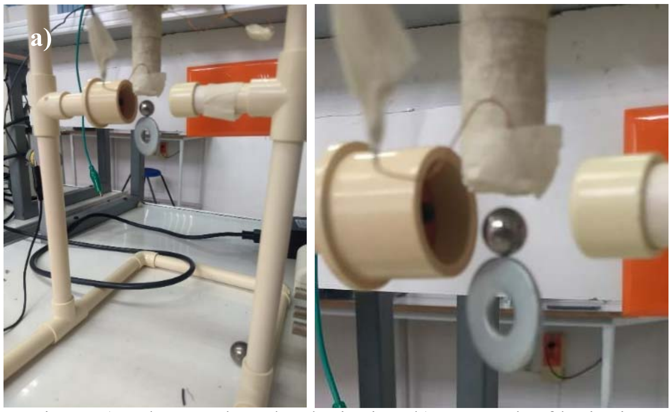

Fig. 7. a) Sphere and washer levitating, b) approach of levitating sphere.

IV. CONCLUSIONS

In this study a mathematical model and electromagnetic levitation system control was designed and implemented with a degree of freedom using a Proportional-Integral-Derivative controller. The system is not linear and sub-actuated, this, it is ideal to be used as a prototype for the stability model in non-linear control. The obtained values for the system plant and the pid controller give local stability at the adjustment point of 4 mm with the object tested over the coil. It is worth considering that the used values for resistances R1, R2 and R4 are close to those calculated, since, when they are implemented by variable low precision resistances, in the design criteria there is a slight error at a stationary position.
The system was designed for a levitation of approximately 4 mm, in accordance with the inertial mass of the metallic sphere weighing 24.75 g, and can support an increase in weight up to 6 g.

Future work we include the design and fabrication of a material other than PVC, looking for the appropriate position of the optoelectronic sensor, and with this to make tests with different inertial masses and ferrous percentages, in order to analyze the behavior of the system.

V. ACKNOWLEDGMENTS

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